Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

"a porcupine of symbols"

- Meaning? Story? ... Inner Beauty?

Algebra : "Solve for x."

- To solve a problem, give a name to _____ — call it x for example — then write down what we _____ & work from there.
- Like solving a mystery : Student X did not write his/her name on their paper ... used colored pens... writes in cursive...
- Example : Inheritance
  - 10 dirhams (unit of currency)
  - two sons, one daughter
  - each should receive share, but sons' shares should each be twice as much as daughter's (Islamic law)
  - Call daughter's share x.
    Then each son's share is ___.
    In total : $10 = $ ___ $+$ ___ $+$ ___
    Simpler : $10 = $ ___
    Therefore $x = $ ___.
      - Daughter receives ___ dirhams. } [ ]
      - Each son receives ___ dirhams.
  - Solving the problem came down to "_____" the equation $10 = 5x$, i.e. finding a number for x that makes a ___ equation: $10 = 5 \cdot 2$.
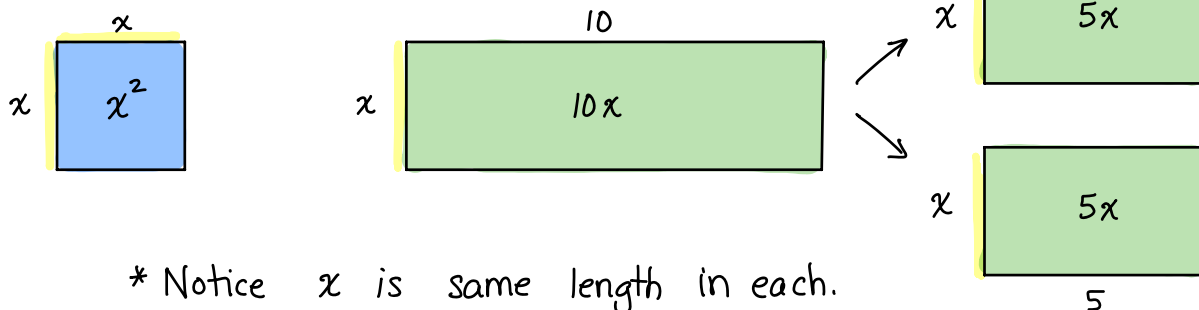
· In general, when dealing with an equation, we may:
 - ___ the ___ number to ___ sides
 - _____ the ___ number from ___ sides
 - _____ ___ sides by the ___ number
 - _____ ___ sides by the ___ number (not ___)
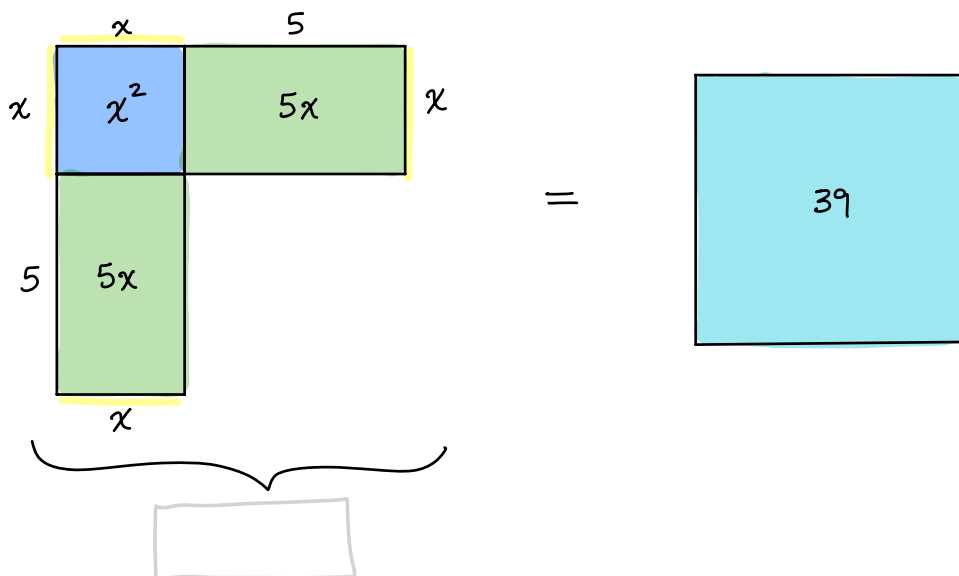
· Quadratic Equations:

$$x^2 + 10x = 39 \quad \ldots \quad 3x^2 - x + 2 = 7 \quad \ldots \quad ax^2 + bx + c = 0$$
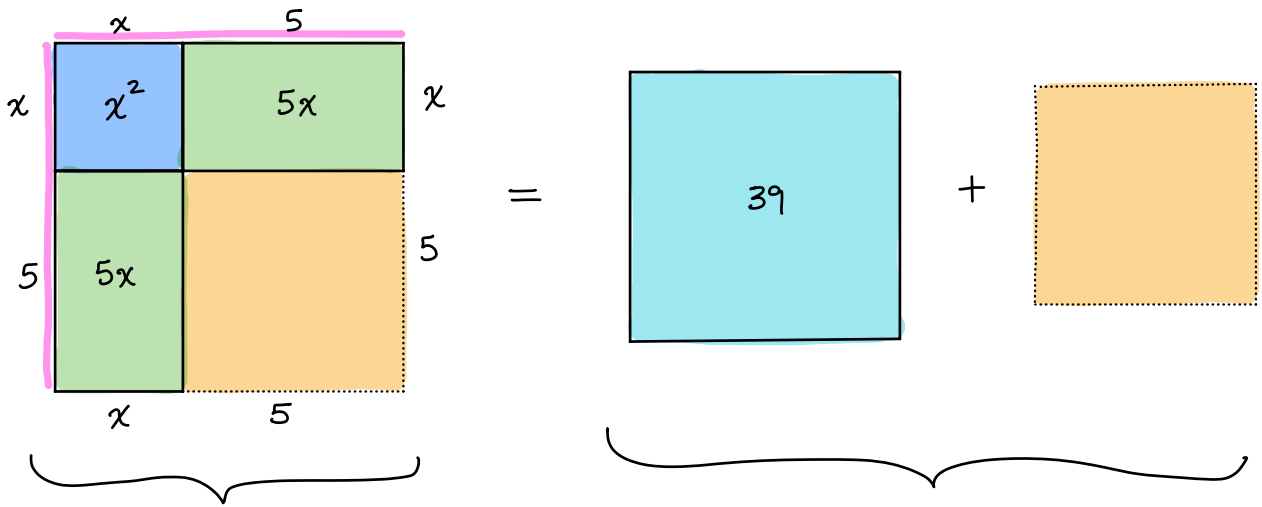
A Geometric Approach : Complete the Square

Solve : $x^2 + 10x = 39$.

* Notice x is same length in each.

Rearrange pieces & set equal to 39:

"Complete" the Square :

$x^2 + 10x + \boxed{}$

$= (\underline{})^2$

$39 + \boxed{} = \underline{}$

$(x+5)^2 = 64$

$(x+5)^2 = (\underline{})^2$

$x = \underline{}$

Check : For $x=3$, do we have $x^2 + 10x = 39$?

Well, $\underline{}$, so $\underline{}$.

From a modern viewpoint, we are missing one of the roots...
· Keep in mind that $(\underline{})^2 = 64$ as well ...

$(x+5)^2 = 64$

$x+5 = 8 \qquad x+5 = -8$

$x = 3$

✓

Using other notation :

$x+5 = \pm\sqrt{64}$

$x = -5 \pm \sqrt{64}$

$x = -5 \pm 8$

Activity # 8

① Solve $x^2 + 6x = 16$ by completing the square.

② Solve $x^2 + 14x = 51$ by completing the square.

③ Further: Suppose B and C are positive numbers.
   Solve $x^2 + 2Bx = C$ by completing the square.

④ Further: Use the quadratic formula to solve the quadratic equations in ① & ②.

See Activity #8 Hand Out for details.